Mesh Simplification

CS418 Interactive Computer Graphics
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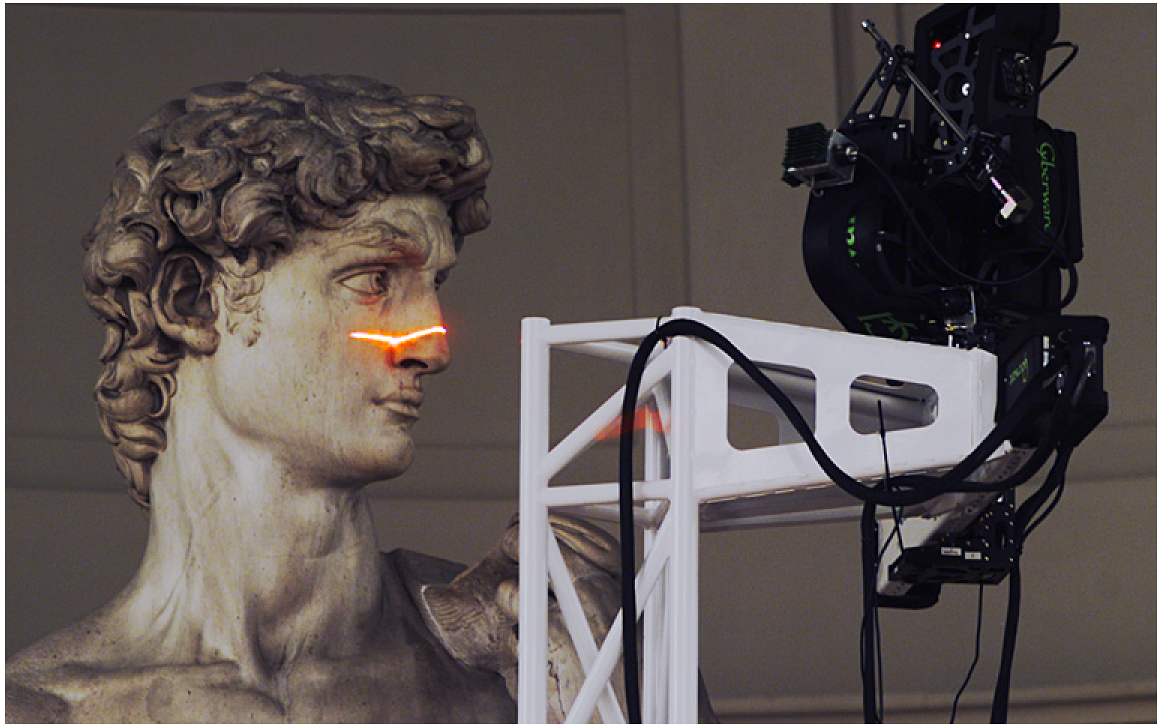
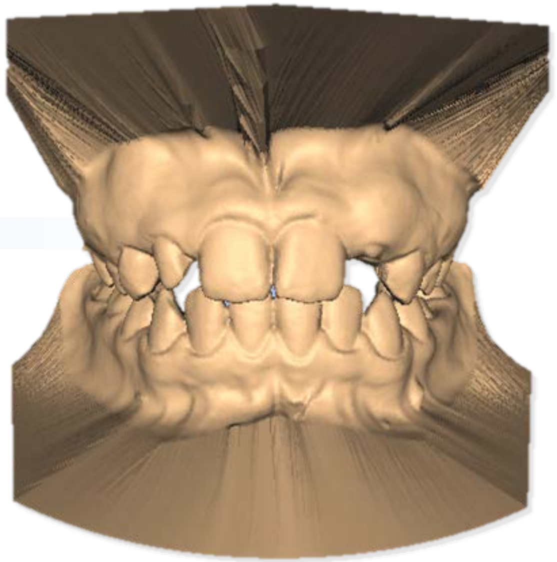
Digital Michelangelo

- In 1998 Marc Levoy takes a sabbatical year in Florence to scan a bunch of Michelangelo sculptures
- David at 1mm resolution
- St. Matthew at 290μm resolution

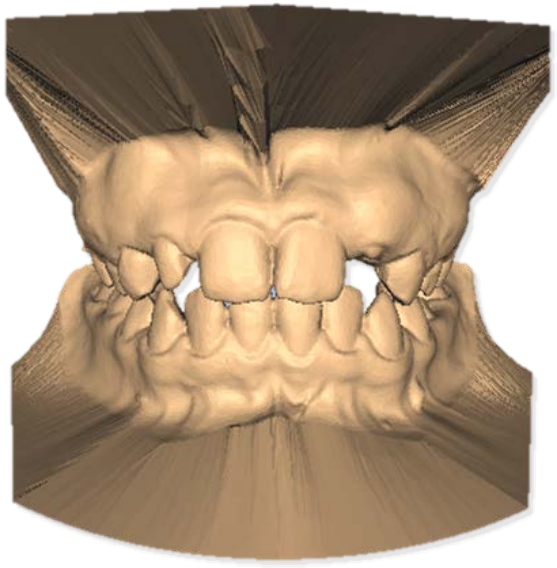


Mesh Simplification

- Meshes often contain more triangles than are necessary for visual fidelity
  - Some surface generation methods run at a fixed resolution regardless of surface detail
  - Meshes might be used on different resolution devices, e.g. cellphone
- Need a method to reduce the number of triangles in a mesh
- Must figure out which triangles to remove while preserving visual shape and appearance



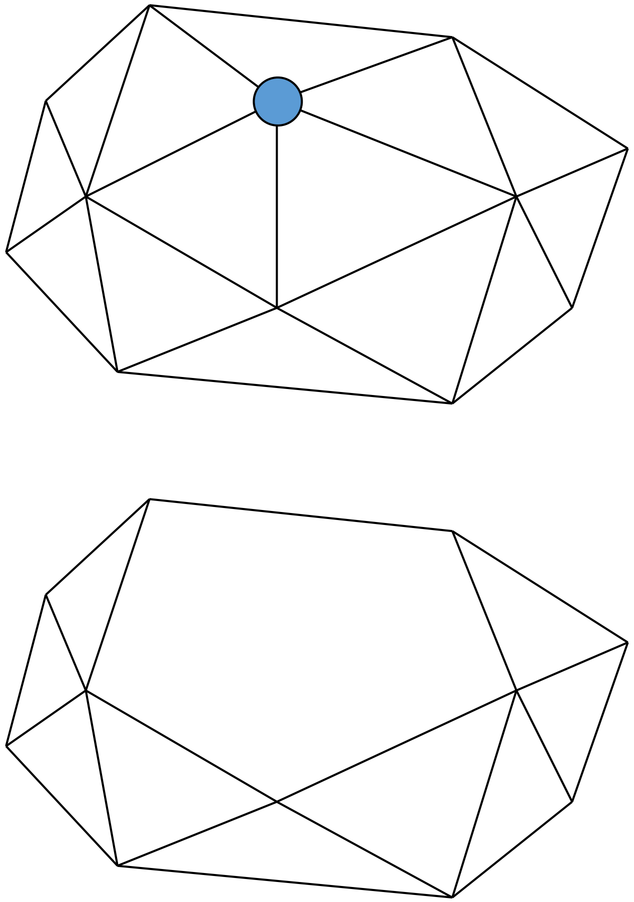
424,376 triangles



60,000 triangles

Vertex Decimation

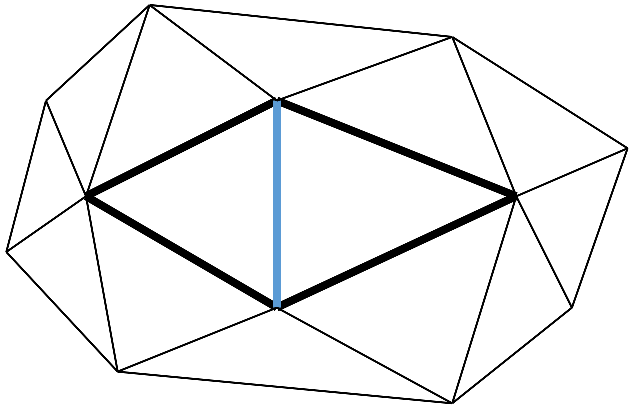
- Removing a vertex turns triangle mesh into polygon mesh

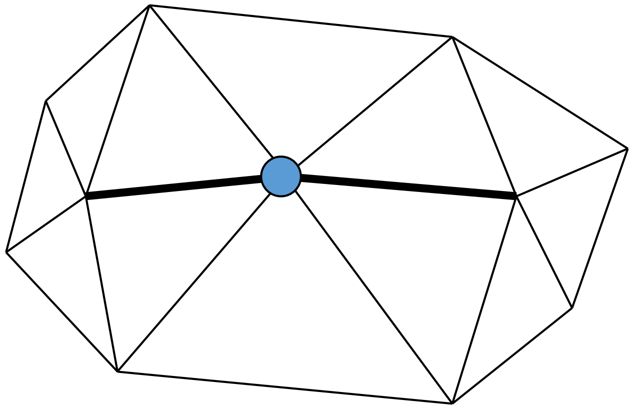


Edge Collapse

- Removing a vertex turns triangle mesh into polygon mesh
- Removing an edge…
  - Merges two vertices into one
  - Removes two faces
  - Mesh still consists of triangles

- Which edges goes first?
- Where should the new vertices go?





Vertex Importance

- Plane equation

  $P_i(x,y,z) = A_i x + B_i y + C_i z + D$

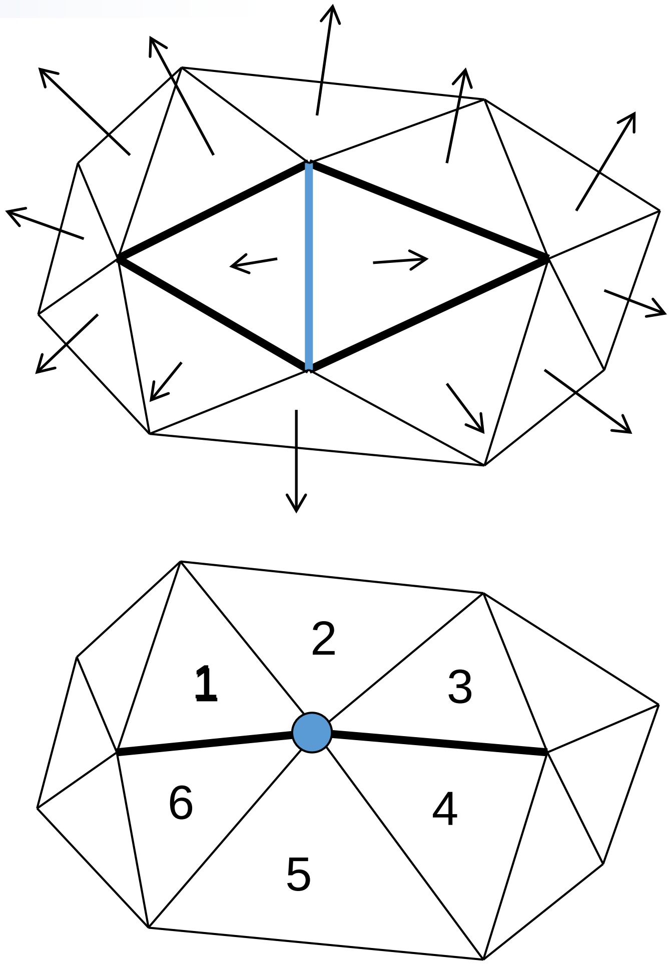
  $P_i(\mathbf{x}) = N_i \cdot \mathbf{x} + D$

- $P_i(\mathbf{x})$ returns signed distance from $\mathbf{x}$ to plane passing through polygon i

- Best position of new vertex position minimizes squared distance to original planes of original polygons

$$QEM(\mathbf{v}) = \sum P_i\,(\mathbf{v})^{\,2}$$

(for adjacent polygons i)



Matrix Representation

- Squared distance from point **x** to plane P

$$P^2(\mathbf{x}) = (Ax + By + Cz + D)^2$$

$$= \begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} A^2 & AB & AC & AD \\ AB & B^2 & BC & BD \\ AC & BC & C^2 & CD \\ AD & BD & CD & D^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \mathbf{x}^T Q \, \mathbf{x}$$

- Sum of squared distances from point **x** to planes P_1 and P_2

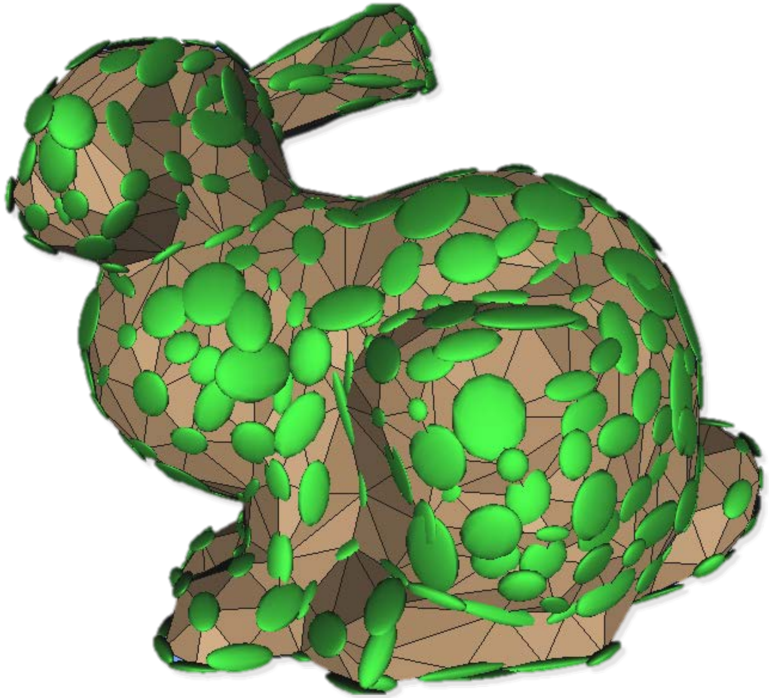
$$P_1^2(\mathbf{x}) + P_2^2(\mathbf{x}) = \mathbf{x}^T Q_1 \, \mathbf{x} + \mathbf{x}^T Q_2 \, \mathbf{x} = \mathbf{x}^T (Q_1 + Q_2) \, \mathbf{x}$$

Quadric Error Metric

- Find Q for each vertex $\mathbf{v}$

$$Q(\mathbf{v}) = \sum Q_i$$

(for adjacent polygons i)

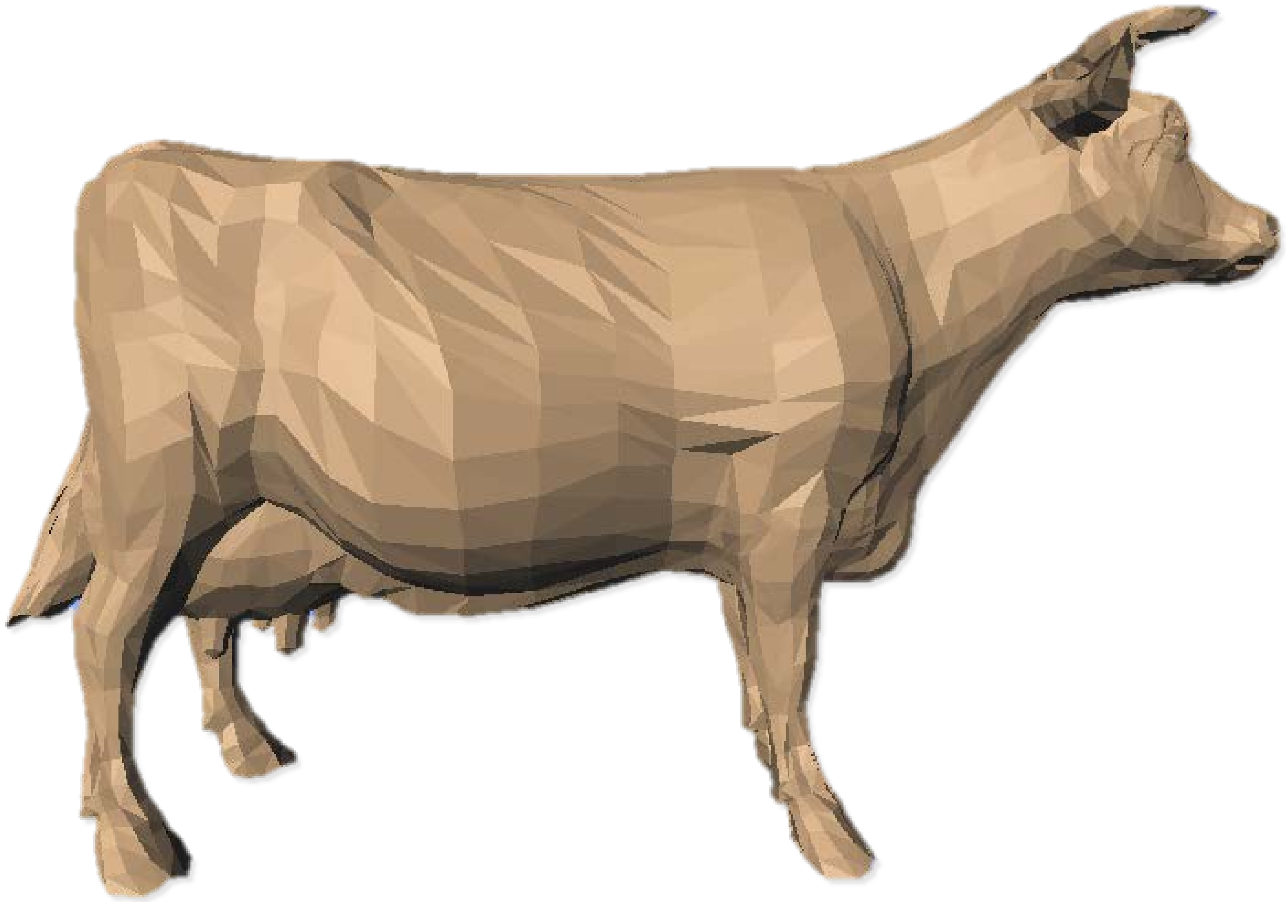


- Edge collapse edges whose vertices $\mathbf{v}_1,\mathbf{v}_2$ have least $Q(\mathbf{v}_1) + Q(\mathbf{v}_2)$

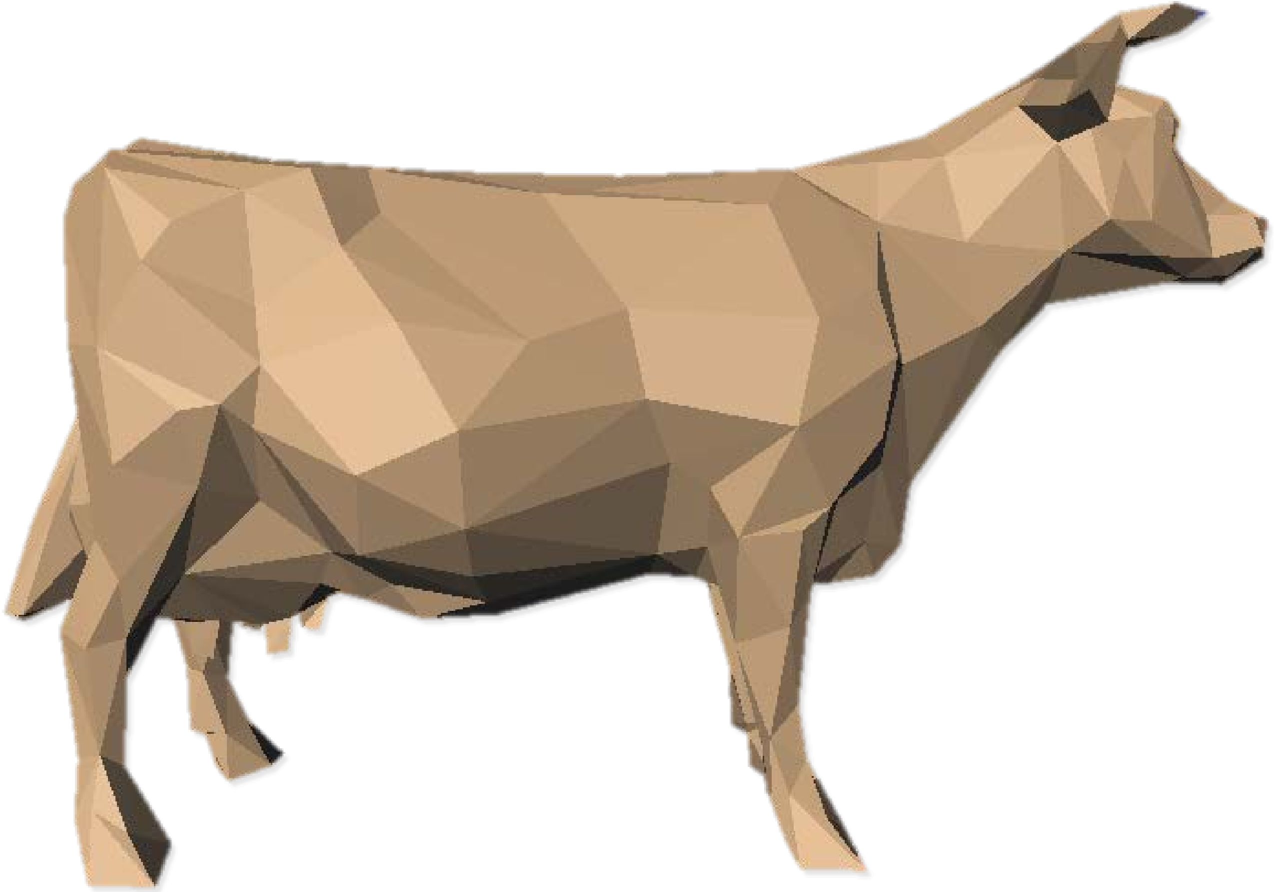
- New vertex $\mathbf{v}_{12}$ can be at $\mathbf{v}_1$ or $\mathbf{v}_2$ or optimally at …

$$\mathbf{v}_{12} = -\begin{bmatrix} A^2 & AB & AC \\ AB & B^2 & BC \\ AC & BC & C^2 \end{bmatrix}^{-1} \begin{bmatrix} AD \\ BD \\ CD \end{bmatrix}$$

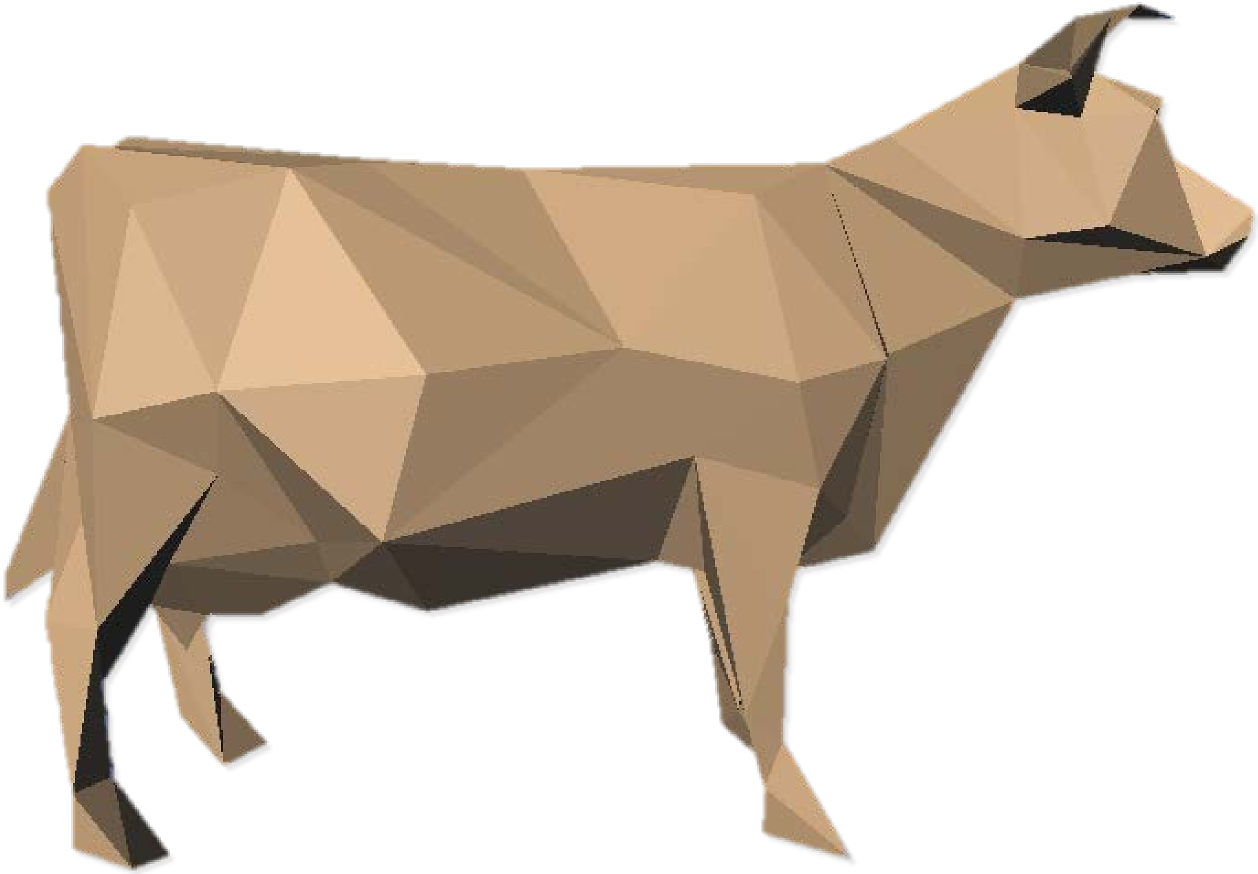
Examples



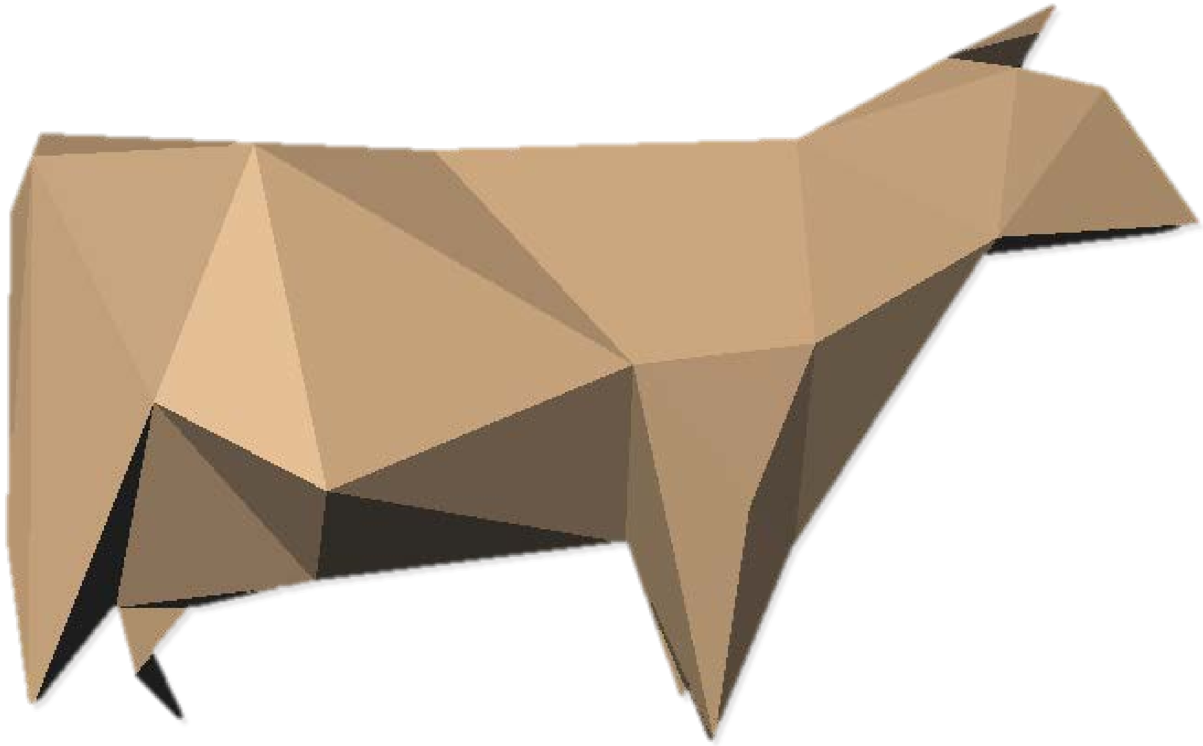
Examples



Examples



Examples



Examples